Int. J. Heat Mass Transfer. Vol. 36, No. 3, pp. 765–773, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00
© 1993 Pergamon Press Ltd

Thermal theory of the spiral heat exchanger

TH. BES and W. ROETZEL

Institut für Thermodynamik, Universität der Bundeswehr, P.O. Box 70 08 22,
D-2000 Hamburg 70, F.R.G.

(*Received* 20 *December* 1991)

Abstract—An analytical investigation of heat transfer in a counterflow spiral plate heat exchanger is the subject of this study. To evaluate the thermal performance of this heat exchanger, a new dimensionless criterion number CN is proposed. For any heat capacity rate ratio and for an arbitrary (even) number of turns one uniform, universal and simple formula is developed to calculate the mean temperature difference correction factor F of a spiral plate heat exchanger: $F = \ln(1+CN^2)/CN^2$. The accuracy of the theory increases with the growing number of channels.

1. INTRODUCTION

ESSENTIAL advantages of spiral plate heat exchangers (SHE) include high thermal effectiveness, compact design and very little inclination to fouling due to only one cross-section for each fluid (see Fig. 1).

The thermal theory of SHE, compared to theories of heat exchangers with other flow arrangements, is insufficiently represented in the literature. Because of the wide application of SHE in industry, more attention should be paid to this exchanger, to its theory of operation and especially to developing simple but sufficiently exact forms for the calculation of its effectiveness if possible. In the present paper, the authors attempt to accomplish the above.

In order to provide satisfactory information on the thermal performance of SHE, as required by a designer, it is necessary to develop adequate formulas and prescriptions that will allow to take into account its thermodynamic limits. Particularly, this limit refers to the maximum of effectiveness. The special thermal behaviour of SHE, namely the characteristic maximum of effectiveness occurring with growing NTU to very high values (10 and more), was noticed for the first time on the basis of exact theory in ref. [1] and then confirmed in refs. [2–4].

The problem of the thermal calculation of SHE has been treated in the literature analytically [1, 5–7], numerically [8], numerically with some experiments [9] and analytically with some heuristic attempts [10, 11].

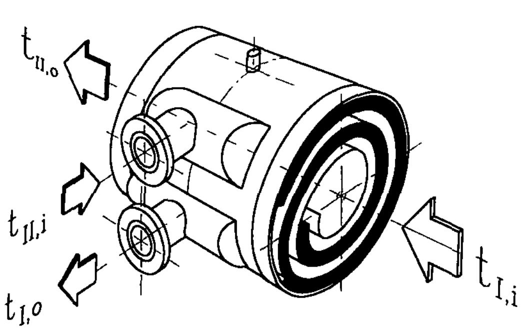

FIG. 1. View of counterflow spiral plate heat exchanger.

All these solutions have more or less disadvantages due to:

- lack of sufficient accuracy either for a small number of turns or for large NTUs and for the reason that this accuracy is estimated only qualitatively [5, 6];
- too arduous computational procedures for engineering design purposes [1, 7, 8];
- insufficient theoretical records [10, 11];
- necessity of iterative adjustment for the initial vector of temperatures by using the Runge–Kutta numerical method of integration [9, 10].

2. IDENTIFICATION OF THE PROBLEM AND FORMULATION OF TASKS

2.1. *Basic assumptions*

The problem is solved under the general assumptions valid for heat exchangers and well known from literature [12, 13]. But the additional assumptions which are typical for SHEs have to be supplied or some standard assumptions of particular importance to the problem should be emphasized:

- Due to different heat transfer conditions, SHE was divided into three parts: the innermost part with two channels where heat flows only through one wall, the middle part with turns, which usually occupy the main volume of exchanger performing its main duty where heat penetrates both walls, and the outermost parts with two channels where heat is transferred only through one wall.
- Flow is in countercurrent.
- Arrangement of fluids' flow in the exchanger is as shown in Fig. 2.
- The shape of the spiral is optionally assumed to be the spiral of Archimedes.

NOMENCLATURE

A_c	cross-sectional area of flow channel, $h_o b'$ [m^2]
A_o	total heat transfer surface area [m^2]
b'	channel spacing (optionally chosen as unit of length) [m]
C	mean heat capacity rate, $\sqrt{(C_I C_{II})}$ [W K^{-1}]
CN	criterion number, $(\sqrt{R}+1/\sqrt{R})NTU\sqrt{(\pi A_c/A_o)}$
F	log mean temperature difference correction factor
h_o	height of exchanger
k	overall heat transfer coefficient [W m^{-2}K^{-1}]
n	number of channels equal to double number of turns
NTU	number of transfer units (mean value), kA_o/C
P	effectiveness, dimensionless temperature change
q	heat flux $\dot{Q}$, related to angle φ (see Fig. 2), $d\dot{Q}/d\varphi$ [W rad^{-1}]
R	heat capacity rate ratio for counterflow, C_I/C_{II}
r	dimensionless radius, r' real radius, r'/b'
t	dimensionless temperature, t' real temperature of fluid I or II, $(t'-t'_{II,i})/(t'_{I,i}-t'_{II,i})$
x	coordinate proportional to distance measured along main spiral, $(\psi/2)(\sqrt{R}+1/\sqrt{R})r$.

Greek symbols

$\Delta(r)$	local temperature difference of fluids flowing on both sides of the main spiral, $t_I(r)-t_{II}(r)$
$\delta(r)$	local temperature difference of fluids flowing on both sides of the side spiral, $t_I(r+1)-t_{II}(r-1)$
Θ	mean temperature difference
μ	auxiliary parameter in equations (6), (8), $(2/\psi)(\sqrt{R}-1/\sqrt{R})/(\sqrt{R}+1/\sqrt{R})^2$
φ	angle in coordinate system (r, φ) (see Fig. 3)
ψ	cross-sectional number of transfer units (mean value), $2\pi kA_c/C$
ω	auxiliary parameter in equations (6), (8), $(\sqrt{R}+1/\sqrt{R})/2$.

Subscripts

i	inlet
o	outlet
I	fluid I
II	fluid II.

- Fluids are completely mixed in the radial and axial directions within the flow channel. Thus, in cross-section chosen at a fixed angle, the temperature changes step by step from channel to channel (see Fig. 3).
- In a single channel the fluid temperature is a function of the angle φ or radius r of the Archimedes' spiral only.
- Hot fluid enters the exchanger in the centre of the apparatus and cold fluid flows in at the outermost channel.
- The number of channels (coils) in SHE is even.†
- Flows of fluid and heat are steady.
- Distances between walls of both channels with hot and cold fluids are equal.
- Influence of outermost turn is considered approximately: it is analysed in the same manner as turns in the middle part of SHE.
- Overall heat transfer coefficient k is constant throughout the exchanger.
- There are no heat losses to the environment.
- Distance studs are not taken into account.

The theory presented in this paper performs the conditions which are fulfilled in the middle part of the exchanger. Therefore, the higher the number of

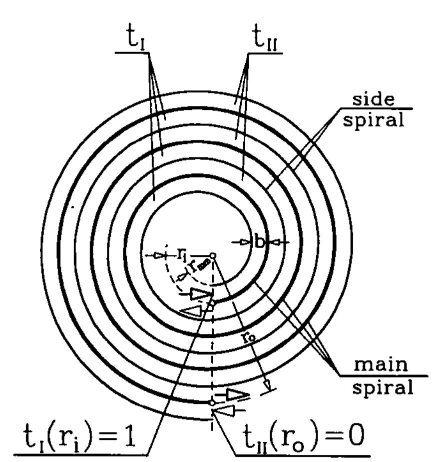

FIG. 2. Arrangement of flows in countercurrent spiral heat exchanger.

† The theory itself is valid for both even and odd numbers of channels. However, the accessible data set based on the exact theory [1,2] which has been used as a reference level to evaluate the accuracy of present theory was produced for an even number of channels.

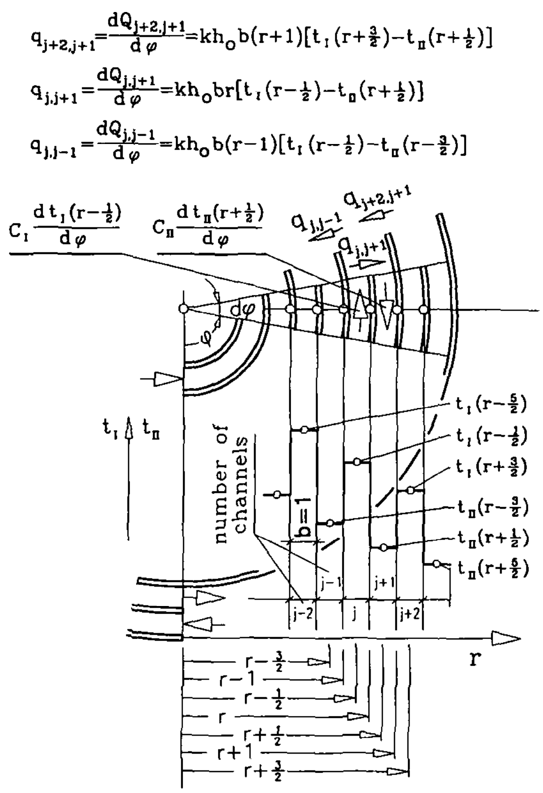

FIG. 3. Temperatures and components of energy balance in elementary wedge of SHE.

channels in SHE the better this theory will render properties of the physical model of the apparatus.

All considerations carried out in this paper refer to the quantities which are expressed in dimensionless forms and listed in the nomenclature.

2.2. *Aim of the paper*

The purpose of the present paper is to provide a simple theory of countercurrent spiral plate heat exchanger and at the same time to develop a straightforward formula for the log mean temperature difference correction factor F which is usually applied to the thermal design of these exchangers. The theory should render the special feature of SHE, i.e. the maximum of its effectiveness for very high quantities of *NTU*.

3. ANALYTICAL FORMULATION OF THE PROBLEM

3.1. *Object of investigations*

The temperatures of fluids flowing on both sides of a partition wall in a heat exchanger vary considerably. The mean value of their difference is one of the most important factors characterizing qualitatively any heat exchanger. This difference will be an object of search.

In SHE there are two walls twisted spirally, called the 'main' and the 'side' spiral which separate fluids. Thus, it is useful to carry out the thermal analysis of temperature distribution using two different functions which represent the local temperature difference Δ and δ, defined as the surplus of temperature of hot fluid over fluid temperatures on both sides.

These differences of temperature depend upon each other; however, for convenience they will be separately derived as two different quantities and will later be combined with each other.

3.2. *Components of energy balance equations*

The components of heat fluxes passing through the wall may be represented by the expressions shown in Fig. 3, in which t_I and t_{II} are the fluid temperatures to be determined.

Choosing as a positive direction of φ, the flow direction of C_I, one obtains for the part of the channel between the radii $r-1$ and r

$$-C_I\frac{dt_I(r-\frac{1}{2})}{d\varphi} = q_{j,j-1}+q_{j,j+1} \tag{1}$$

and for the part of the channel between the radii r and $r+1$

$$-C_{II}\frac{dt_{II}(r+\frac{1}{2})}{d\varphi} = q_{j,j+1}+q_{j+1,j+2}. \tag{2}$$

It is convenient to change slightly the notation of temperatures. The function of temperature $t_I(r-\frac{1}{2})$ and $t_{II}(r+\frac{1}{2})$ will be further denoted as $t_I(r)$ and $t_{II}(r)$, respectively. Applying the formula for the spiral of Archimedes $r' = r'_{\min}+b'\varphi/\pi$ or using reduced radii $r = r_{\min}+\varphi/\pi$, the differential $d\varphi$ in equations (1), (2) is: $d\varphi = \pi\, dr$. Bearing in mind the new notation of the temperatures and introducing formulas for: $q_{j,j-1}$, $q_{j,j+1}$, $q_{j+1,j+2}$ from Fig. 3 into equations (1) and (2) the following system of differential difference equations is obtained:

$$-C_I\frac{dt_I(r)}{\pi\, dr} = kh_o b'\{r[t_I(r)-t_{II}(r)] + (r-1)[t_I(r)-t_{II}(r-2)]\}$$

$$-C_{II}\frac{dt_{II}(r)}{\pi\, dr} = kh_o b'\{r[t_I(r)-t_{II}(r)] + (r+1)[t_I(r+2)-t_{II}(r)]\} \tag{3}$$

with the boundary conditions for the fluid temperatures at the inlet: $t_I(r_i) = 1$ and $t_{II}(r_o) = 0$, according to Figs. 1 and 2.

3.3. *Energy balance equations for the main spiral*

By simple transformation and using the symbols for ψ, Δ, δ and R listed in the nomenclature, equations (3) can be simplified to

$$2\sqrt{R}\frac{dt_I(r)}{dr}+\psi r\Delta(r)+(r-1)\psi\delta(r-1) = 0$$

$$\frac{2}{\sqrt{R}}\frac{dt_{II}(r)}{dr}+\psi r\Delta(r)+(r+1)\psi\delta(r+1) = 0. \tag{4}$$

Let one transform the set of equations (4) as follows. Divide the first equation and multiply the second equation by $\sqrt{R}$, respectively. Subtract the second equation from the first. Thus

$$2\frac{d\Delta(r)}{\psi\, dr} - \left(\sqrt{R} - \frac{1}{\sqrt{R}}\right) r\Delta(r) + \frac{(r-1)\delta(r-1)}{\sqrt{R}} - (r+1)\delta(r+1)\sqrt{R} = 0. \quad (5)$$

Bearing in mind that $r \gg 1$, expand the function $f(r \pm \varepsilon) = (r \pm \varepsilon)\delta(r \pm \varepsilon)$ up to the third term, according to Taylor's general theorem:

$$f(r \pm \varepsilon) \approx f(r) \pm \frac{df(r)}{1!\, dr}\varepsilon + \frac{d^2 f(r)}{2!\, d^2 r}\varepsilon^2$$

and put finally $\varepsilon = 1$.

Substituting the last formula in equation (5) and rearranging the equation to cumulate all constant parameters in front of functions Δ, δ or their derivatives, one arrives at the relation

$$2\frac{d\Delta(r)}{\psi\, dr} - (\sqrt{R} - 1/\sqrt{R})\left\{r\Delta(r) + r\delta(r) + \frac{d^2[r\delta(r)]}{2\, dr^2}\right\} - (\sqrt{R} + 1/\sqrt{R})\frac{d[r\delta(r)]}{dr} = 0.$$

To reduce the number of parameters let one introduce the new independent variable x and the auxiliary parameter μ defined in the nomenclature. This allows us to rewrite the last equation with required functions Δ and δ as follows:

$$\frac{d\Delta(x)}{dx} - \mu x[\Delta(x) + \delta(x)] - \frac{d[x\delta(x)]}{dx} - \omega^2\psi^2\mu\frac{d^2[x\delta(x)]}{2\, dx^2} = 0. \quad (6)$$

The boundary conditions are: $\Delta(x_i) = \Delta_i$, $\delta(x_i) = \delta_i$.

3.4. *Energy balance equations for the side spiral*

To formulate the second equation with the required functions $\Delta(x)$ and $\delta(x)$, it is necessary to write the basic system of difference–differential equations (4) again, but for the new independent variable r as coordinate, that is, with moved steps: one forward $r+1$ and one backward $r-1$ for the first and second equation in system (4), respectively.

This procedure is equivalent to the previous notation of energy balance equations, for the set of radii $r-1$, r and $r+1$, where r is the reference radius. However, now the set of energy balance equations relates to the radius r lying on the second spiral, i.e. on the side spiral (see Fig. 2), whereas the previous notation has been referred to the main spiral.

This procedure has a clear geometrical and physical interpretation. If the first set of equations (4) was selected optionally on any radial cross-section of SHE, the second refers to the cross-section and radius r which is turned 180° around SHE's axis in reference to prior. Thus

$$2\sqrt{R}\frac{dt_I(r+1)}{dr} + (r+1)\psi\Delta(r+1) + \psi r\delta(r) = 0$$

$$\frac{2}{\sqrt{R}}\frac{dt_{II}(r-1)}{dr} + (r-1)\psi\Delta(r-1) + \psi r\delta(r) = 0. \quad (7)$$

The last system of energy balance equations should be treated in a similar manner to equations (5), but this time functions $f(r \pm 1) = (r \pm 1)\Delta(r \pm 1)$ are expanded into a Taylor's series. As a consequence one arrives at the relation

$$\frac{d\delta(x)}{dx} - \mu x[\Delta(x) + \delta(x)] + \frac{d[x\Delta(x)]}{dx} - \omega^2\psi^2\mu\frac{d^2[x\Delta(x)]}{2\, dx^2} = 0. \quad (8)$$

The boundary conditions are the same as for equation (6).

3.5. *Local temperature differences in SHE*

Parameter ψ is small in comparison to NTU_I or NTU_{II}, which rarely exceed 10. In engineering practice the value of NTU would increase up to about 4 or 5. Because $A_c/A_o \ll 1$, in almost all cases $\psi \ll 1$.

This assumption ($0 < \psi \ll 1$) is physically justified for a high number n of channels for which cases the following theory is valid.

Due to the small quantity ψ the terms with second derivative in equations (6) and (8) having the parameters $\omega^2\psi^2\mu \sim \psi$ in front can be neglected. Thus, the system of energy balance equations (6) and (8) simplifies to

$$\frac{d\Delta(x)}{dx} - \mu x[\Delta(x) + \delta(x)] - \frac{d[x\delta(x)]}{dx} = 0$$

$$\frac{d\delta(x)}{dx} - \mu x[\Delta(x) + \delta(x)] + \frac{d[x\Delta(x)]}{dx} = 0. \quad (9)$$

The boundary conditions stay as for equations (6) and (8).

The purpose of the above procedure was to transform the difference–differential equations (6) and (8) into the differential equations (9) (in any order), without inconvenient differences as independent variables.

Solution of the system in equation (9) is achieved in the following way: subtracting both equations and integrating yields

$$\Delta(x) - \delta(x) - x[\Delta(x) + \delta(x)] = \Delta_i - \delta_i - x_i(\Delta_i + \delta_i) = \text{const.}$$

The sum of function $[\Delta(x) + \delta(x)]$ will be searched for because it is useful in calculations of the effectiveness P_I. Adding both equations (9) and replacing $\Delta(x) - \delta(x)$ using the last relation gives

$$\frac{\mathrm{d}\{(1+x^2)[\Delta(x)+\delta(x)]\}}{\mathrm{d}x}-2\mu x[\Delta(x)+\delta(x)]$$
$$= -[\Delta_i-\delta_i-x_i(\Delta_i+\delta_i)]. \quad (10)$$

An additional assumption refers to the radius r_{min}, which should be small enough to allow neglect of the term with $x_i = \psi\omega r_i$ in the last equation.

As a consequence of the assumptions regarding the high number $n(\Delta_i \approx \delta_i)$ and the small quantity of the minimal radius r_{min}, the constant on the right-hand side of equation (10) can be ignored.

The solution of equation (10) is readily obtainable. The final function $\Delta(x)+\delta(x)$ is

$$\Delta(x)+\delta(x) = (\Delta_i+\delta_i)\frac{G_i}{(1+x^2)^{1-\mu}} \quad (11)$$

where $G_i = (1+x_i^2)^{1-\mu}$ is a constant computed for radius r_i at the inlet to the exchanger which does not differ much from 1.

On the basis of equation (11) and previous considerations it is easy to see that the function $\Delta(x)$ for the local temperature difference between fluids separated by the main partition wall is proportional to the auxiliary function $G_\mu(x)$ which is defined as follows:

$$G_\mu(x) = \frac{1+x}{1+x^2}(1+x^2)^\mu. \quad (12)$$

The function $G_\mu(x)$ is plotted against the independent variable x in Fig. 4.

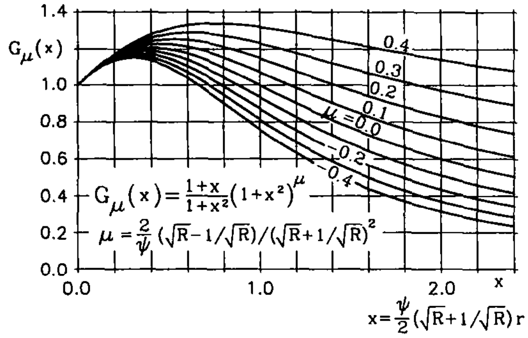

FIG. 4. Auxiliary function $G_\mu(x)$ proportional to the local temperature difference $\Delta(x)$ of fluids separated by the main spiral.

4. EFFECTIVENESS OF SHE

The effectiveness P_1 for SHE can be calculated according to a general formula

$$P_1 = -\int_{r_i}^{r_o}\mathrm{d}t_1 = 1-t_1(r_o)$$

where r_i and $r_o = r_i+n$ are the radii of bent walls: the smallest and the largest, respectively, through which heat is transferred between both fluids (see Fig. 2) and $t_1(r_i) = 1$.

Further consideration will refer to the first form in equations (4) which allows one to express the derivative of temperature as

$$\frac{\mathrm{d}t_1(r)}{\mathrm{d}r} = -\frac{1}{2\sqrt{R}}[\psi r\Delta(r)+(r-1)\psi\delta(r-1)] \quad (13)$$
$$\approx -\frac{1}{2\sqrt{R}}\psi r[\Delta(r)+\delta(r)].$$

Integration of equation (13) together with solution (11) leads to

$$P_1 \approx \Delta_i E_\mu G_i/(2\psi\omega^2\sqrt{R}) \quad (14)$$

where

$$E_\mu = [(1+x_o^2)^\mu-(1+x_i^2)^\mu]/\mu. \quad (15)$$

Putting expression (14) into equity $\Delta_i+P_{II} = 1$ or $\Delta_i+P_1R = 1$, which are well known from the literature [12, 13], one finds

$$\Delta_i = 1/(1+eR), \quad \text{and} \quad P_1 = e/(1+eR) \quad (16)$$

where $e = E_\mu G_i/(2\psi\omega^2\sqrt{R})$.

Introducing definition (15) into a formula for e

$$e = \{[(1+x_o^2)/(1+x_i^2)]^\mu-1\}(1+x_i^2)/(R-1)$$

and applying the last result to equation (16) after simple manipulations one gets the form for effectiveness P_1 of SHE:

$$P_1 = \frac{1-\exp\{\mu\ln[(1+x_o^2)/(1+x_i^2)]\}}{1-R\exp\{\mu\ln[(1+x_o^2)/(1+x_i^2)]\}+(R-1)x_i^2/(1+x_i^2)}. \quad (17)$$

The exponent in form (17) consists of two components which should be analysed step by step.

For further analysis some auxiliary parameters are needed.†‡ First component μ

† Quotient of areas A_o/A_c: the definitions listed in the nomenclature lead to the following expression:

$$2\pi A_c/A_o = \psi/NTU. \quad (a)$$

For any spiral with the arrangement of flows shown in Fig. 2 and for known function $r(\varphi)$ a form

$$\frac{A_o}{\pi A_c} = \frac{1}{\pi}\left(\int_{r_i}^{r_o=r_i+n} r\,\mathrm{d}\varphi+\int_{r_i+1}^{r_o-1} r\,\mathrm{d}\varphi\right)$$

is in force.

Marked intervals of integration can be easily seen in Fig. 2. Applying the differential $\mathrm{d}\varphi = \pi\,\mathrm{d}r$, valid for the spiral of Archimedes, and integrating yields

$$\frac{A_o}{\pi A_c} = r_o^2-r_i^2-r_o-r_i = (r_o+r_i)(r_o-r_i-1)$$
$$= (n-1)(n+2r_i). \quad (b)$$

‡ Another auxiliary relation, which refers to cross-sectional areas in SHE is useful

$$r_o^2-r_i^2 = n(n+2r_i) = \left(1+\frac{1}{n-1}\right)A_o/(\pi A_c). \quad (c)$$

$$\mu = \frac{2}{\psi}(\sqrt{R}-1/\sqrt{R})/(\sqrt{R}+1/\sqrt{R})^2$$
$$= (R-1)NTU_I/CN^2$$

where

$$CN = (\sqrt{R}+1/\sqrt{R})NTU\sqrt{(\pi A_c/A_o)}. \quad (18)$$

This number is characteristic for SHE and therefore it is proposed to recognize CN as the dimensionless criterion number for the countercurrent spiral heat exchanger.

The second term of exponent with ln function contains an independent variable which can be expressed as follows:

$$(1+x_o^2)/(1+x_i^2) = 1+(x_o^2-x_i^2)/(1+x_i^2)$$

and

$$\frac{x_o^2-x_i^2}{1+x_i^2} = CN^2\frac{1+\frac{1}{n-1}}{1+x_i^2}.$$

If the cross-sectional number of transfer units ψ or reduced radius r_i at inlet are so small that quantity x_i can be neglected in comparison to 1 or x_o then for realistic quantities of ratio R the term $(R-1)\times x_i^2/(1+x_i^2)$ can also be ignored.

For the ratio $R = 1$, this term has to be left out for any quantity of minimal radius r_i, but for very high or very small quantities R this is not allowed due to the parameter ω which, in accordance with definition $x = \psi\omega r$, includes this variable. Now, let one look closer at formula (17). It is easy to notice that equation (17) without the already ignored term has the same structure as the formula for effectiveness P_I in the true countercurrent heat exchanger.

The exponent in equation (17) is equal to the product $(R-1)NTU_I$ and the log mean temperature difference correction factor

$$F = \ln\left[1+\left(1+\frac{1}{n-1}\right)CN^2\right]\Big/CN^2. \quad (19)$$

For a high number of channels n and under the simplifications mentioned in Section 1, the form (19) for the F correction factor in SHE can be written as

$$F = \ln(1+CN^2)/CN^2. \quad (20)$$

The function F is plotted against CN in Fig. 5.

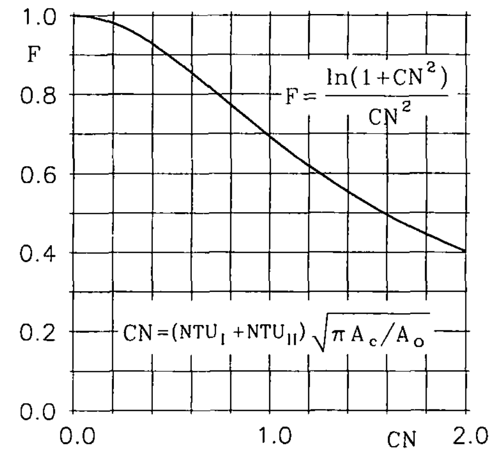

FIG. 5. Log mean temperature difference correction factor F vs criterion number CN for SHE.

5. CRITERION NUMBER

Returning to form (18), it is easy to note that the criterion number CN can be rewritten using both values of NTU_I and NTU_{II}. This yields the formula

$$CN = (NTU_I+NTU_{II})\sqrt{(\pi A_c/A_o)}. \quad (21)$$

From basic literature [12, 13] it is known that for the true cocurrent heat exchanger the sum of NTU_I and NTU_{II} is an argument in the formula for its effectiveness. Therefore, the last relation indicates that for a high number of channels there could be some connection between the countercurrent SHE and the true cocurrent heat exchanger.

This similarity was used by Martin *et al.* [11] to propose the countercurrent cascade set of $(n/2)$ true cocurrent heat exchangers as the model which is thermally equivalent to SHE; however, his proposal had no theoretical background. The last formula enables proof of Martin's former suggestion.

In the previous paper [4] a similar straightforward formula for the F correction factor was suggested. The achievement from ref. [4] has been derived analytically for the case $R = 1$ and extended to other quantities of R by using a hypothesis proposed by Roetzel. This hypothesis was also successfully applied to thermal analysis of other flow arrangements, e.g. cross-flow [14]. The formula for the criterion number CN_g achieved in ref. [4] is

$$CN_g = 2\sqrt{(NTU_I NTU_{II})}\sqrt{(\pi A_c/A_o)}. \quad (22)$$

The difference between the forms (21) and (22) developed here and in ref. [4] lies in the manner of calculating the mean value of NTU:

- In the present paper NTU is calculated as the arithmetic mean value from NTU_I and NTU_{II}. The formula (21) has a good theoretical background, but it fails for ratios $R = 0$ and $R \to \infty$, where factor F has to be equal to one. It is so because this theory was developed under conditions which exclude the large and small quantities of ratio R.
- In ref. [4] the NTU is calculated as the geometric mean of NTU_I and NTU_{II}. In contrast to the present analysis the form (22) works correctly ($F = 1$) for both limiting values of ratio: $R = 0$ and $R \to \infty$.

The arithmetic is greater than the geometric mean value if $NTU_I \neq NTU_{II}$, and both mean values are equal if $NTU_I = NTU_{II}$.

For the general applicability of equation (20) another mean value of NTU_I and NTU_{II} would be useful, which value should fall between the arithmetic and geometric mean and turn to zero for $R \to 0$ or ∞. In the region $NTU_I \approx NTU_{II}$ it should approach the arithmetic mean value.

These conditions are fulfilled by the logarithmic mean value and consequently the following formula could be recommended for the criterion number:

$$CN^* = \frac{NTU_I - NTU_{II}}{\ln(NTU_I/NTU_{II})}\sqrt{(\pi A_c/A_o)}. \quad (23)$$

Taking in the formula for CN the log mean value of NTU instead of the arithmetic mean is not a consequence of any thermal analysis, but only the proposal which works well in these domains of value R ($R \to 0$, $R \to \infty$) where the previous derivation loses its validity due to undeterminate quantities of the product: $\omega = (\sqrt{R} + 1/\sqrt{R})/2$ and coefficients in Taylor's series (see equation (6)).

6. COMPARISON OF APPROXIMATE ACHIEVEMENTS WITH EXACT THEORY OF SHE

6.1. *Reference level for comparison*

Many simplifications were made on the way from equation (5) to the relation (21). They include:

- ignorance of the real boundary conditions by considering the outermost and innermost turns in the same way as turns in the bulk part of SHE;
- replacement of temperature functions with moved independent arguments, by their differentials (according to Taylor's theorem);
- neglecting the components of the temperature functions with terms having cross-sectional number of transfer units with higher power;
- exclusion from analysis of the cases with extremely high or small heat capacity rate ratios ($R = 0$ and $R \to \infty$);
- neglecting of the term $(R-1)/(1+1/x_i^2)$ in the denominator of equation (17).

All these steps, although done for good physical or mathematical reasons, could weaken the reliability of the approximate theory.

For this reason the approximate achievement presented in equations (18) and (20) should be verified.

To demonstrate how this approximate theory fits in with results of the exact thermal theory of SHE, the evaluation of effectiveness P_I must be done for the whole field of solution. This is possible under the condition that there is access to the set of data with effectiveness of the counter flow SHE for different values of R, for different number of turns, for different values of minimal radii, and all of the calculations were done without any simplifications in comparison to a physical model.

The exact theory of SHE with complete mathematical accuracy in reference to the physical model is a subject of the study in ref. [2]. There, the effectiveness and the mean temperature difference have been calculated for heat capacity rate ratio: $R = 0.0$; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.0; 1.2; 1.4; 1.6; 1.8; 2.0; 2.5; 3.0; 3.5; 4.0; 4.5; 5.0; 6.0; 8.0 and 10.0, for the number of turns n and minimal radius r_i in SHE, denoted as $(n; r_i)$: (4; 1), (4; 2), (4; 3), (4; 4), (6; 2), (6; 3), (6; 4), (8; 4), (8; 5), (10; 5), (12; 5), (14; 5), (16; 6), (18; 5), (20; 5), (22; 5), (24; 5), (26; 5), (28; 5), (30; 5), (32; 5), (34; 5), (36; 5), (38; 5) and (40; 5).

As a geometrical basis for construction of SHE it was optionally assumed that the spiral was traced out as an involute from an equilateral triangle. This assumption differs slightly from the assumption taken into account in the present paper (see Section 2), but it fits in with the way of manufacturing the SHEs.

The following proof of the actually presented theory for different parameters R and NTU in SHE is, in the main, a comparison of effectiveness P_I achieved on the basis of the formulas expressed by equations (18), (20) with the data for P_I taken from ref. [2]. Actually, the evaluations of results for both theories are done in three different ways according to the kind of parameters which are compared.

The effectiveness P_I of any heat exchanger can be calculated from the formula

$$P_I = \frac{1 - \exp[(R-1)NTU_I F]}{1 - R\exp[(R-1)NTU_I F]} \quad (24)$$

in which the F correction factor takes into account the deviation from true counterflow. Solving for F yields

$$F = \frac{\ln[1 + (R-1)/(1/P_I - 1)]}{(R-1)NTU_I}. \quad (25)$$

This will be the basis formula for the evaluation of present achievement in reference to the exact theory of SHE.

6.2. *Evaluation with regard to different number of turns*

The theory developed in this paper has been carried out under the assumption that the number of turns in SHE is high. But despite this limitation it is of interest to know, too, how adequate the theory is for a very small number of turns. An answer to this question is possible by using form (20). Thus, having the exact values: P_I (taken from ref. [2]) and factor F from equation (25) one can reverse the function in equation (20) to find the value of hypothetical number CN_o which gives this exact value P_I. On the other hand, the real value CN could be calculated directly from equation (18). A quotient of both may be a reference level to demonstrate how close the approximate achievements of this paper are to the exact theory. Let one denote this quotient by f and pass to the limit at $R = 1$ and $\psi \to 0$:

$$f = \lim_{\psi \to 0} CN/CN_o. \quad (26)$$

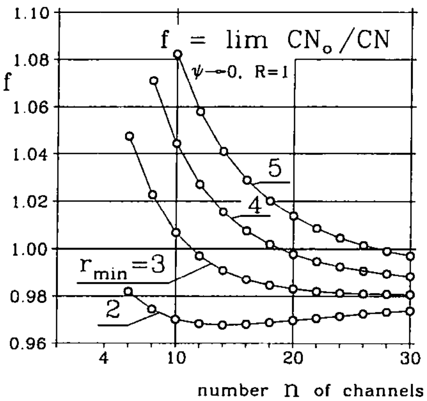

FIG. 6. Factor $f = \lim_{\psi\to 0, R=1} CN_o/CN$ (CN_o on the basis of exact theory [2] in comparison to present theory of SHE) as function of number of channels n for different radii r_{min}.

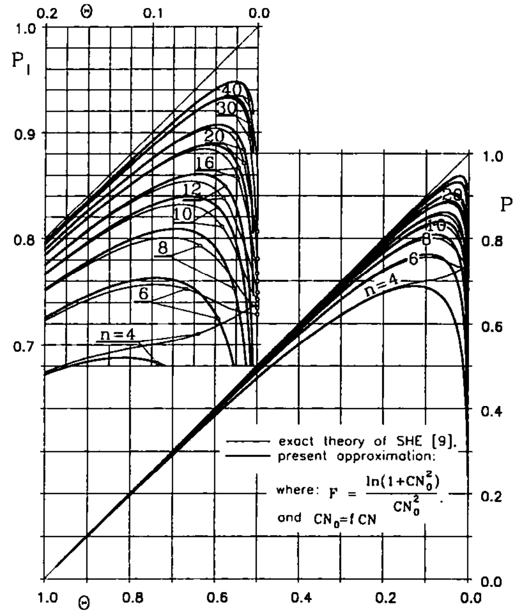

FIG. 7. Effectiveness P_I of spiral heat exchanger vs mean temperature difference Θ for heat capacity rate ratio $R = 1$ and different number n of channels (for $n = 4, 6, 8$, $r_{min} = 2, 3, 4$, respectively, and for $n \geqslant 10$, $r_{min} = 5$).

In Fig. 6 a set of points is shown which represents values of the f factor against number of turns in SHE.

Indeed, an influence of the number of turns on the F factor has already been taken into account in the formula (18) by quotient A_o/A_c. Thus, the factor f can be understood as some further improvement of the criterion number regarding the minimal radius and the number of turns in SHE. With a growing number of turns the factor f approaches unity, so that it can be assumed as equal to 1.

6.3. *Evaluation for very high NTUs and* $R = 1$

If the values of NTU are small, average or even high, up to 10 the effectiveness of SHE grows with increase of NTUs and differs slightly from the effectiveness of the true counterflow exchanger; however, not small enough to be ignored.

But for very high NTU, >10–30, and $n \geqslant 6$, as investigated in the refs. [1, 2], the effectiveness P_I achieves its maximum and further, and with increase of NTU starts to decrease, approaching finally the constant value (0.7–0.9) for unlimited NTUs. It is due to thermal conjunction between fluids which takes place throughout the side spiral (see Fig. 2), causing some return of heat flow and oscillations of the local temperature difference [2]. Equation (18) together with form (20) renders very well these properties of SHE, i.e. the maximum of P_I which is pointed out in Fig. 7. But for extremely high values the difference between both theories tends to grow. Finally, values P_I calculated from equation (21) approach zero. For these quantities of NTU the theory loses its applicability.

6.4. *Evaluation with regard to heat capacity rate ratio R*

The analysis and comparison of results performed on the basis of both theories allows one to state that a key to the evaluation of SHE lies in the knowledge of function P_I for ratio $R = 1$ (see ref. [4]).

To verify quantitatively the present theory and disadvantageous cases of numbers $n = 4$, 8, 12 and 16 were analysed.

In the diagram, mean temperature difference Θ vs effectiveness P_I, the most important area with respect to practical application is the region shaped as a triangle and limited by the lines $\Theta = 0$, $P_I = 1$ and $NTU_I = 1$ (see Fig. 8).

For a given small number of turns n where the conditions of the theory are not fulfilled, the lines with constant R are drawn on the basis of data from ref. [2]. The same procedure is repeated for data calculated using equation (20) under the conditions of equations (20) and (21). In the diagrams (Fig. 8) the lines with constant deviation of effectiveness P, expressed in percentages, are shown

7. CONCLUSIONS

The problem of thermal analysis in countercurrent spiral heat exchangers was solved on the basis of the energy balance equations by using the regular mathematical transformations; however, with some approximations, resulting in the straightforward formula for the log mean temperature difference correction factor: $F = \ln(1+CN^2)/CN^2$.

All thermal and geometrical parameters of SHE are combined in only one new dimensionless number CN which could be recognized as criterion number for SHE.

The verification of the present approach allows testification that the approximate theory under consideration fits with the exact theory [2] expressed in

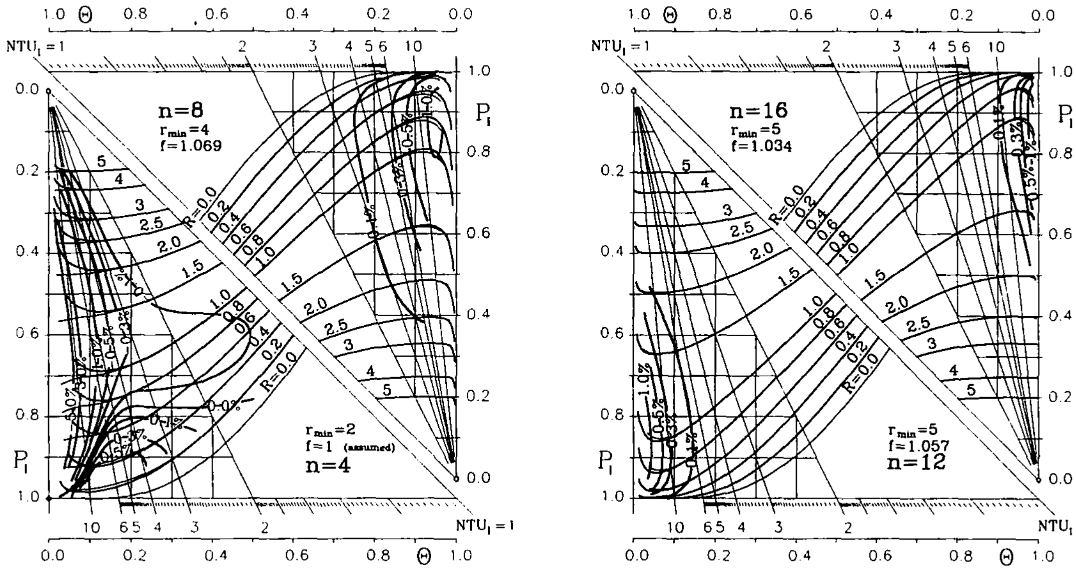

FIG. 8. Comparison of exact theory (thin lines) with present approximation (thick lines) for different number of channels: (a) $n = 4$, (b) $n = 8$, (c) $n = 12$, (d) $n = 16$, and $r_{min} = 2$, 4, 5 and 5, respectively. Quantities of equal difference $(P_{I,approx} - P_{I,exact})$ are marked in % with extra thick lines.

terms of effectiveness P_I. The new theory appears useful for the design of countercurrent SHE.

Acknowledgement—The authors would like to thank the Deutsche Forschungsgemeinschaft for the financial support of this research project.

REFERENCES

1. Th. Bes, Eine Methode der thermischen Berechnung von Gegen- und Gleichstrom-Spiralwärmeaustauschern, *Wärme- und Stoffübertragung* **21,** 301–309 (1987).
2. Th. Bes and W. Roetzel, Distribution of heat flux density in spiral heat exchangers, *Int. J. Heat Mass Transfer* **35,** 1331–1347 (1992).
3. M. Strenger, S. Churchill and W. Retallik, Operational characteristic of a double-spiral heat exchanger for the catalytic incineration of contaminated air, *Ind. Engng Chem. Res.* **29,** 1977–1984 (1990).
4. Th. Bes and W. Roetzel, Approximate theory of spiral heat exchanger. *Design and Operation of Heat Exchangers, Proceedings of the Eurotherm Seminar No. 18*, Hamburg, 27 Feb.–1 March 1991 (Edited by W. Roetzel, P. J. Heggs and D. Butterworth), pp. 223–232. Springer, Berlin (1991).
5. J. Madejski, *Teoria Wymiany Ciepla* (*Theory of Heat Exchange*), pp. 381–386. PWN, Warszawa—Poznan (1963).
6. T. Zaleski and W. Krajewski, Metoda obliczania wymiennikow spiralnych (Method of calculation spiral exchangers), *Inz. Chem.* (*Chem. Engng*) **II,** 1, 35, 35–51 (1972).
7. J. P. Cieslinski and T. Bes, Analytical heat transfer studies in spiral plate heat exchangers, Reprints of XVI Congr. of Refrigeration, Institut International du Froid (I.I.F.), Paris, pp. 67–72 (1983).
8. W. Nowak, Obliczenia rekuperatorow z uwzglednieniem strat do otoczenia (Calculating of heat exchangers considering heat losses to environment), *Zeszyty Naukowe Politechniki Szczecinskiej* (*Scientific Journals Tech. Uni. of Stettin*) No. 114 (1969).
9. R. A. Buonopane and R. A. Troupe, Analytical and experimental heat transfer studies in a spiral plate heat exchanger, *IV International Heat Transfer Conference*, Vol. 1, He 2.5, Paris (1970).
10. K. Chowdhury, H. Linkmeyer, M. K. Bassiouny and H. Martin, Analytical studies on the temperature distribution in spiral plate heat exchangers, *Chem. Engng Process.* **19,** 183–190 (1985).
11. H. Martin, K. Chowdhury, H. Linkmeyer and K. M. Bassiouny, Straightforward design formulae for efficiency and mean temperature difference in spiral plate heat exchanger, *VIII International Heat Transfer Conference*, Vol. 6, 2793–2797, San Francisco (1986).
12. W. Roetzel, *Berechnung von Wärmebertragern* (Chapter C of *VDI-Wärmeatlas*, 4th edn). VDI, Düsseldorf (1984).
13. E. S. Gaddis, D. B. Spalding and J. Taborek, *Heat Exchanger Theory* (Vol. I of *Heat Exchanger Design Handbook*). VDI GmbH, Hemisphere, Washington (1986).
14. B. Spang and W. Roetzel, Approximate equations for the design of cross-flow heat exchangers. *Design and Operation of Heat Exchangers, Proceedings of the Eurotherm Seminar No. 18*, Hamburg, 27 Feb.–1 March 1991 (Edited by W. Roetzel, P. J. Heggs and D. Butterworth), pp. 125–134. Springer, Berlin (1991).